Class 10-CBSE-Mathematics

Chapter-3

PAIR OF LINEAR EQUATIONS IN TWO VARIABLES

Example of Linear equation in two variables x and y.

ax + by + c = 0,

where a, b and c are real numbers, and a and b are not both zero.

This condition can also be denoted by the condition $a^2 + b^2 \neq 0$.

Solution of above equation is a pair of values, one for x and the other for y, which makes the two sides of the equation equal.

Consider the equation ***2x + 3y = 5.***

- let us substitute x = 1 and y = 1 in the left-hand side (LHS) of the equation::

  2x + 3y = 5.

  Then LHS = 2(1) + 3(1) = 2 + 3 = 5, which is equal to the right-hand side (RHS) of the equation.

  Therefore, x = 1 and y = 1 is a solution of the equation 2x + 3y = 5

- Now let us substitute x = 1 and y = 7 in the equation 2x + 3y = 5.

  Then, LHS = 2(1) + 3(7) = 2 + 21 = 23 *which is not equal to the RHS*. *Therefore, x = 1 and y = 7 is not a solution of the equation.*

- ***Geometrical Interpretation of the above solutions***
  - ✓ *point (1, 1) lies on the line representing the equation*

    2x + 3y = 5,
  - ✓ *point (1, 7) does not lie on it.*

- ***every solution of the equation is a point on the line representing it.***

- *each solution (x, y) of a linear equation in two variables,*

  ax + by + c = 0,

  corresponds to a point on the line representing the equation, and vice versa

- *The general form for a pair of linear equations in two variables x and y is*

$$a_1x + b_1y + c_1 = 0$$

And

$$a_2x + b_2y + c_2 = 0$$

,

where a_1 *,are all real numbers*

and $a_1^2 + b_1^2 \neq 0$*,* $a_2^2 + b_2^2 \neq 0$

Some examples of pair of linear equations in two variables:

- ✓ $2x + 3y - 7 = 0$ and $9x - 2y + 8 = 0$
- ✓ $5x = y$ and $-7x + 2y + 3 = 0$
- ✓ $x + y = 7$ and $17 = y$

A pair of linear equations in two variables can be represented as two straight lines, considered together

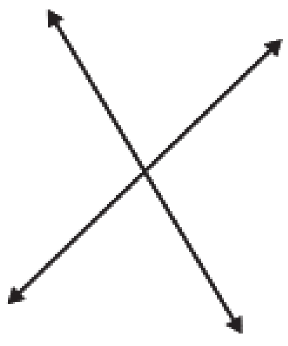
Given two lines in a plane, only one of the following three possibilities can happen:

(i) The two lines will intersect at one point.

(ii) The two lines will not intersect, i.e., they are parallel.

(iii) The two lines will be coincident.

All the possibilities in (i), (ii) & (iii) above are displayed in Fig. 3.1:



(a)

Two Lines Intersect

(b)

The two lines will not intersect, i.e., they are parallel.

(c)

The two lines are coincident.

Example 1 :

Akhila goes to a fair with Rs 20 and wants to have rides on the Giant Wheel and play Hoopla. Represent this situation algebraically and graphically (geometrically)

Solution:

The pair of equations formed is : $y = \frac{1}{2}x$

i.e., $x - 2y = 0$ ………………………………(1)

$3x + 4y = 20$ …………………………… (2)

Let us represent these equations graphically.
For this, we need at least two solutions for each equation.
We give these solutions in Table 3.1

Table 3.1

x	0	2
$y = \frac{x}{2}$	0	1

(i)

x	0	$\frac{20}{3}$	4
$y = \frac{20-3x}{4}$	5	0	2

(ii)

When one of the variables is zero, the equation reduces to a linear equation in one variable, which can be solved easily.

Putting x = 0 in Equation (2), we get 4y = 20, i.e., y = 5.
Similarly, putting y = 0 in Equation (2), we get

$$3x = 20 \ i.e. \ x = \frac{20}{3}$$

But as $\frac{20}{3}$ is not an integer it cannot plotted in a graph paper.
Hence choose y = 2 which gives x = 4, an integral value.
Plot the points A(0, 0), B(2, 1)
and P(0, 5), Q(4, 2), corresponding
to the solutions in Table 3.1.
Now draw the lines AB and PQ,
representing the equations
x – 2y = 0 and 3x + 4y = 20, as
shown in Fig. 3.2.

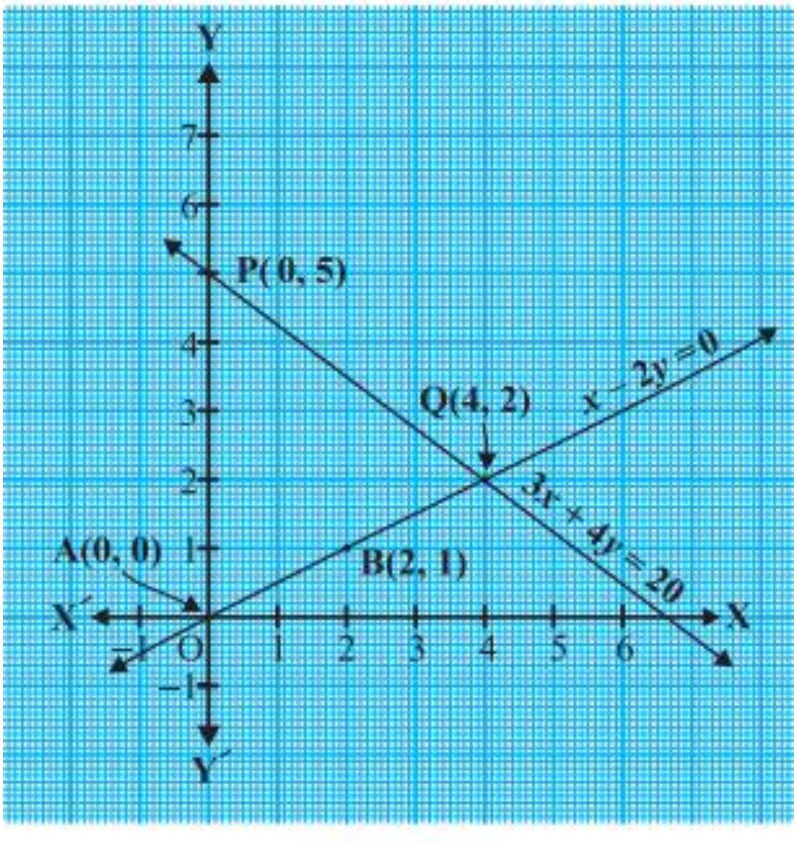


Fig. 3.2

In Fig. 3.2, observe that the two lines representing the two equations are intersecting at the point (4, 2).

Example 2 :
Romila went to a stationery shop and purchased 2 pencils and 3 erasers for Rs 9. Her friend Sonali saw the new variety of pencils and erasers with Romila, and she also bought 4 pencils and 6 erasers of the same kind for Rs 18. Represent this situation algebraically and graphically

Solution :
Let us denote the cost of 1 pencil by Rs x and one eraser by Rs y. Then the algebraic representation is given by the following equations:

$$2x + 3y = 9 \quad \ldots\ldots(1)$$
$$4x + 6y = 18 \quad \ldots\ldots(2)$$

To obtain the equivalent geometric representation, we find two points on the line representing each equation. That is, we find two solutions of each equation **Table 3.2.**

Table 3.2

x	0	4.5
$y = \frac{9-2x}{3}$	3	0

(i)

x	0	3
$y = \frac{18-4x}{6}$	3	1

(ii)

We plot these points in a graph paper and draw the lines. We find that. both the lines coincide (see Fig. 3.3). This is so, because, both the equations are equivalent, i.e., one can be derived from the other.

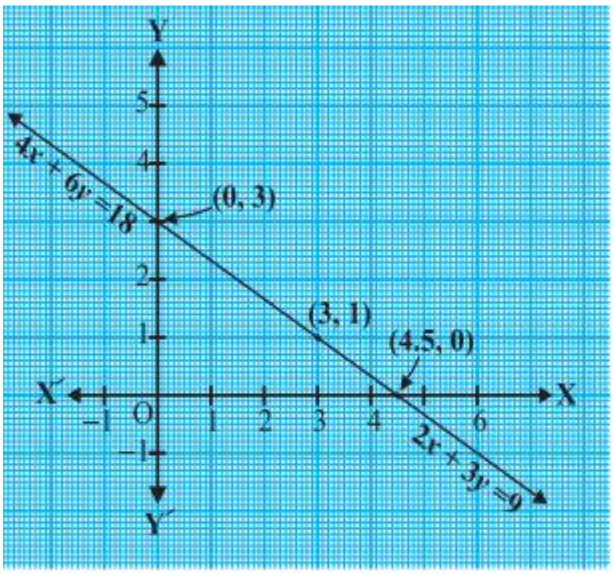


Fig. 3.3

Example 3 :

Two rails are represented by the equations.

$$x + 2y - 4 = 0 \text{ and } 2x + 4y - 12 = 0.$$

Represent this situation geometrically.

Solution :

Two solutions of each of the equations
x + 2y – 4 = 0 ………………………………………………………….(1)
2x + 4y – 12 = 0……………………………………………………….. (2)
are given in Table 3.3

Table 3.3

x	0	4
$y = \frac{4-x}{2}$	2	0

(i)

x	0	6
$y = \frac{12-2x}{4}$	3	0

(ii)

To represent the equations graphically, we plot the points R(0, 2) and S(4, 0), to get the line RS and the points P(0, 3) and Q(6, 0) to get the line PQ.

We observe in Fig. 3.4, that the lines do not intersect anywhere, i.e., they are parallel.

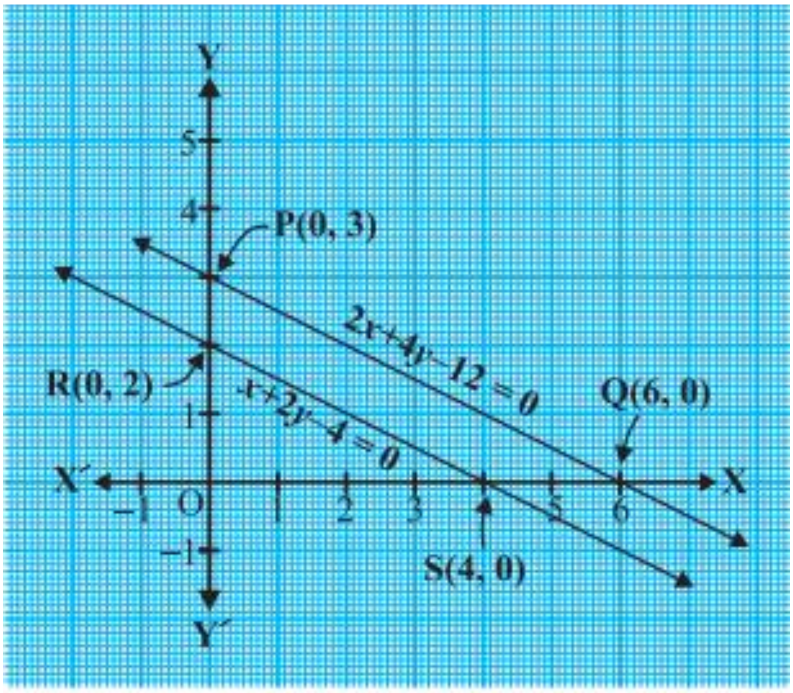


Fig. 3.4

EXERCISE 3.1

1. Aftab tells his daughter, "Seven years ago, I was seven times as old as you were then. Also, three years from now, I shall be three times as old as you will be." (Isn't this interesting?) Represent this situation algebraically and graphically.

Solution:

Let the present age of Aftab be x and present age of daughter be y.

Hence, **seven years ago**.

Age of Aftab = x — 7

Age of daughter = y — 7

Hence, as per the given condition,

(x — 7) = 7(y — 7)

=> x - 7 = 7y — 49

=> x - 7y = -42..............................(i)

Three years later,

Age of Aftab = x + 3

Age of daughter = y + 3

Hence, as per the given condition,

{x + 3) = 3(y + 3)

=> x + 3 = 3y + 9

=> x - 3y = 6............................(ii)

Hence, equation (i) and (ii) represent given conditions algebraically as:

x - 7y = -42

x - 3y = 6

Graphical Representation:

x - 7y = -42

=> x = -42 + 7y

Two Solutions of this equation are:

x	-7	0
y	5	6

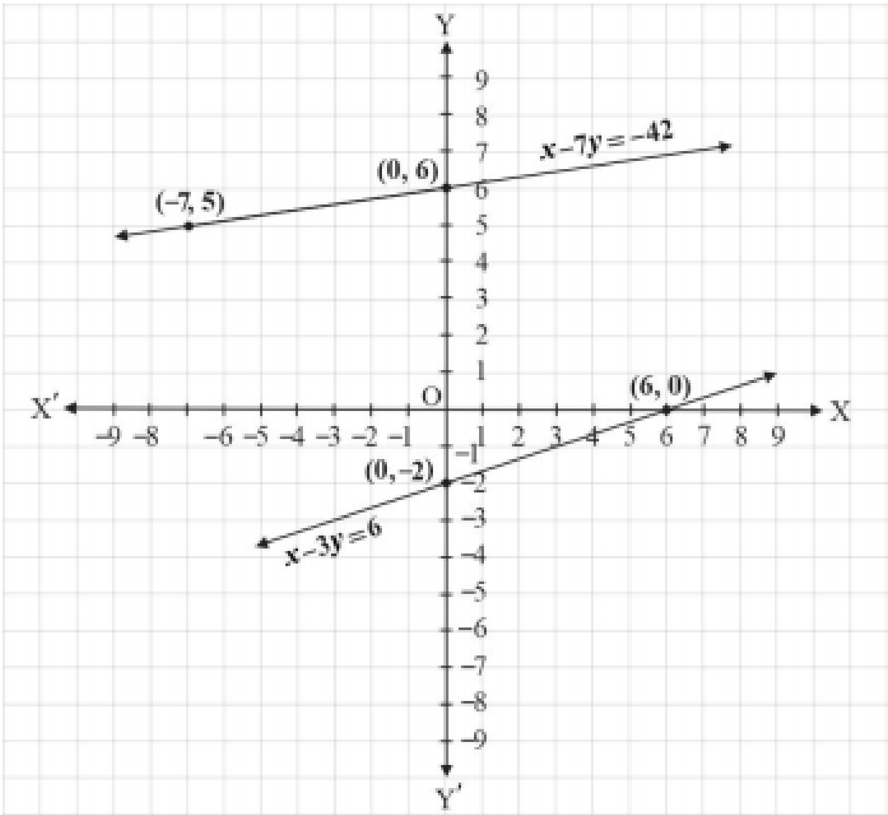
$$x-3y=6$$

$$\Rightarrow x = 6 + 3y$$

Two Solutions of this equation are:

x	6	0
y	0	-2

The graphical representation is as follows:



2. The coach of a cricket team buys 3 bats and 6 balls for Rs. 3900. Later, she buys another bat and 2 more balls of

the same kind for Rs 1300. Represent this situation algebraically and geometrically.

Solution:

Let the price of a bat be ₹ x and a ball be ₹ y.

Hence, we can represent algebraically the given conditions as:

$3x + 6y = 3900$

$x + 3y = 1300$

$3x + 6y = 3900 \Rightarrow x = \frac{3900-6y}{3}$

Two solutions of this equation are:

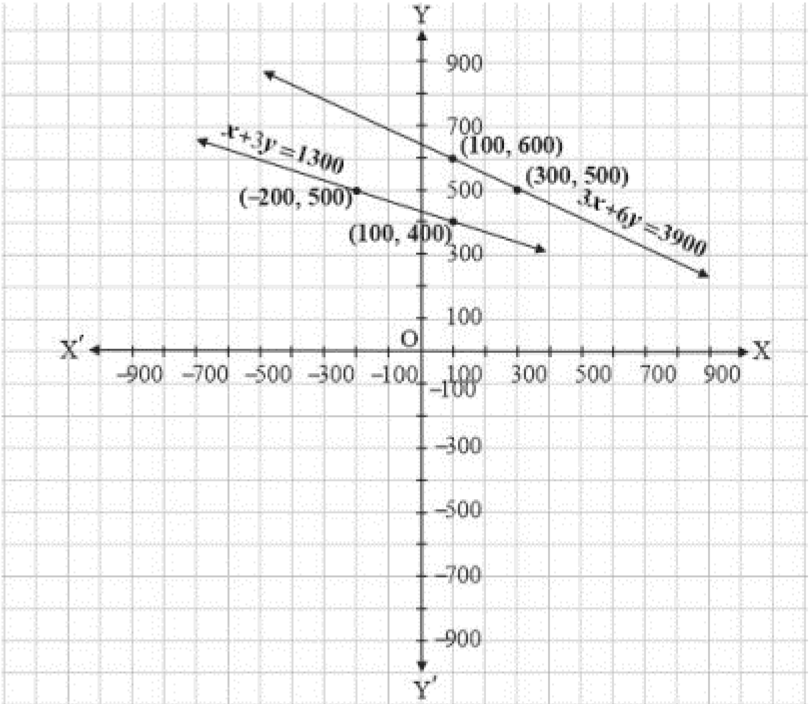
X	300	100
y	500	600

$x + 3y = 1300 \Rightarrow x = 1300 - 3y$

Two solutions of this equation are:

X	100	-200
y	400	500

The graphical representation is as follows:



3. The cost of 2 kg of apples and 1kg of grapes on a day was found to be Rs 160. After a month, the cost of 4 kg of apples and 2 kg of grapes is Rs 300. Represent the situation algebraically and geometrically

Solution:

Let the cost of 1 kg of apples be ₹ x and 1 kg grapes be ₹ y.

The given conditions can be algebraically represented as:

$$2x + y = 160 \qquad (1)$$

$$4x + 2y = 300 \qquad (2)$$

2x + y = 160 =» y = 160 — 2x

Two solutions of this equation are:

x	50	80
y	60	0

$$4x + 2y = 300 \Rightarrow y = \frac{300 - 4x}{2}$$

Two solutions of this equation are:

x	60	75
y	30	0

The graphical representation is as follows: